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Review: Some Remarks on Quantized Lie Superalgebras of Classical Type

Gizem Karaali
Pomona College

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Geer, Nathan (1-GAIT)

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In [Selecta Math. (N.S.) **2** (1996), no. 1, 1–41; MR1403351 (97f:17014)], P. I. Etingof and D. A. Kazhdan constructed a quantization for an arbitrary Lie bialgebra over a field k of characteristic zero. In his earlier work [Adv. Math. **207** (2006), no. 1, 1–38; MR2264064 (2007g:17019)], the author developed a similar theory for the graded case, proving in particular that the Drinfeld-Jimbo type superalgebra $U_h^{\mathrm{DJ}}(\mathfrak{g})$ associated to a classical Lie superalgebra of type A–G, with distinguished Cartan matrix (which was explicitly defined by H. Yamane [Publ. Res. Inst. Math. Sci. **30** (1994), no. 1, 15–87; MR1266383 (95d:17017)] in terms of generators and relations) is isomorphic to the Etingof-Kazhdan quantization of the Lie superalgebra. In this paper, he extends this result to arbitrary Cartan matrices A and thus to arbitrary Drinfeld-Jimbo algebras $U_h^{\mathrm{DJ}}(\mathfrak{g}, A, \tau)$.

This is a very important result in its own right. However, the paper also includes some interesting consequences. The first of these describes all highest weight modules of a Lie superalgebra of type A–G as deformations of appropriate modules of the corresponding Drinfeld-Jimbo type superalgebra. The proof does not automatically follow by a mere superization of the proof of the non-graded version, because the original proof of the non-graded statement makes use of the vanishing of a particular cohomology whose graded counterpart does not vanish in general.

The second interesting corollary is the super version of the Drinfeld-Kohno theorem [T. Kohno, Ann. Inst. Fourier (Grenoble) **37** (1987), no. 4, 139–160; MR0927394 (89h:17030); V. G. Drinfeld, Algebra i Analiz **1** (1989), no. 6, 114–148; MR1047964 (91b:17016)] relating the monodromy of the Knizhnik-Zamolodchikov equations to a representation of the braid group arising from the universal R-matrix of $U_h^{\mathrm{DJ}}(\mathfrak{g}, A, \tau)$.

Reviewed by *Gizem Karaali*

References

1. N. Andruskiewitsch, Lie superbialgebras and Poisson–Lie supergroups, Abh. Math. Sem. Univ. Hamburg 63 (1993) 147–163. MR1227871 (94g:17004)
2. V.G. Drinfeld, Almost cocommutative Hopf algebras, Algebra i Analiz 1 (2) (1989) 30–46 (in Russian): Leningrad Math. J. 1 (2) (1990) 321–342. MR1025154 (91b:16046)
3. V.G. Drinfeld, On quasitriangular quasi-Hopf algebras and on a group that is closely connected with $\mathrm{Gal}(\overline{Q}/Q)$, Algebra i Analiz 2 (4) (1990) 149–181 (in Russian); Leningrad Math. J. 2 (4) (1991) 829–860. MR1080203 (92f:16047)
4. V.G. Drinfeld, On Some Unsolved Problems in Quantum Group Theory, Lecture Notes in Math., vol. 1510, Springer, Berlin, 1992. MR1183474 (94a:17006)
5. V.G. Drinfeld, Quasi-Hopf algebras, Algebra i Analiz 1 (6) (1989) 114–148 (in Russian);

Leningrad Math. J. 1 (6) (1990) 1419–1457. MR1047964 (91b:17016)
6. P. Etingof, D. Kazhdan, Quantization of Lie bialgebras, I, Selecta Math. 2 (1) (1996) 1–41. MR1403351 (97f:17014)
7. P. Etingof, D. Kazhdan, Quantization of Lie bialgebras. VI, preprint, math.QA/0004042, 2000. cf. MR 2002i:17022
8. F. Floreanini, D. Leites, L. Vinet, On the defining relations of quantum superalgebras, Lett. Math. Phys. 23 (1991) 127–131. MR1148504 (92m:17010)
9. N. Geer, Etingof–Kazhdan quantization of Lie superbialgebras, Adv. Math. 207 (2006) 1–38. MR2264064 (2007g:17019)
10. N. Geer, B. Patureau-Mirand, Multivariable link invariants arising from $\mathfrak{gl}(2|1)$ and the Alexander polynomial, J. Pure Appl. Algebra 210 (2007) 238–298. MR2311186 (2008g:57009)
11. N. Geer, B. Patureau-Mirand, Multivariable link invariants arising from Lie superalgebras of type 1, preprint, math.GT/0609034.
12. M. Gould, R.B. Zhang, A. Bracken, Lie bi-superalgebras and the graded classical Yang–Baxter equation, Rev. Math. Phys. 3 (2) (1991) 223–240. MR1121469 (92h:17014)
13. V. Kac, Lie superalgebras, Adv. Math. 26 (1977) 8–96. MR0486011 (58 #5803)
14. C. Kassel, Quantum Groups, Grad. Texts in Math., vol. 155, Springer-Verlag, 1994. MR1321145 (96e:17041)
15. S.M. Khoroshkin, V.N. Tolstoy, Twisting of quantum (super)algebras. Connection of Drinfeld's and Cartan–Weyl realizations for quantum affine algebras, preprint, hep-th/9404036, 1994. cf. MR 98j:81157
16. S.M. Khoroshkin, V.N. Tolstoy, Universal R-matrix for quantized (super)algebras, Comm. Math. Phys. 141 (3) (1991) 599–617. MR1134942 (93a:16031)
17. D. Leites, V. Serganova, Solutions of the classical Yang–Baxter equation for simple superalgebras, Theoret. and Math. Phys. 58 (1) (1984) 26–37. MR0740213 (85m:82020)
18. M. Scheunert, R. Zhang, Cohomology of Lie superalgebras and their generalizations, J. Math. Phys. 39 (9) (1998) 5024–5061. MR1643330 (99f:17025)
19. Y. Su, R.B. Zhang, Cohomology of Lie superalgebras $\mathfrak{sl}_{m|n}$ and $\mathfrak{osp}_{2/2n}$, Proc. London Math. Soc. (3) 94 (1) (2007) 91–136. MR2293466 (2008f:17034)
20. H. Yamane, Quantized enveloping algebras associated with simple Lie superalgebras and their universal R-matrices, Publ. Res. Inst. Math. Sci. 30 (1) (1994) 15–87. MR1266383 (95d:17017)
21. R.B. Zhang, Quantum enveloping superalgebras and link invariants, J. Math. Phys. 43 (4) (2002) 2029–2048. MR1892766 (2003a:17022)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.